AKM and AGN.* Similarly, a line from H to D will pass through the point L.

The point P, where AG and HD intersect is the limiting position. For every triangle within KIJ whose homologous sides are parallel to those of KIJ will have the same relation to KIJ that the latter has to ABC. Therefore the line AG passes through the right angled vertices of all such triangles within KIJ, and HD passes through the middle points of all the bases.

Let D be the origin of coördinates, DC the axis of x and DA the axis of y, then the coördinates of P are easily found to be $x=\frac{1}{5}DC$, $y=\frac{2}{5}DC$.

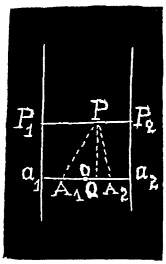
Also solved by L. E. Newcomb, Elmer Schuyler, F. D. Posey, and G. W. Greenwood.

232. Proposed by O VEBLEN, Ph. D., The University of Chicago.

Given two parallel lines a_1, a_2, and two points A_1, A_2, upon a common perpendicular to a_1, a_2 such that A_1 is at the same distance from a_1 as A_2 is from a_2. Let P_1 be the foot of the perpendicular from a point P of the same plane to the line a_1 and P_2 the foot of the perpendicular from P to a_2. Find the locus of P when $\dfrac{PA_1}{PP_1}=\dfrac{PA_2}{PP_2}$.

Solution by J. SCHEFFER, Hagerstown, Md., and A. H. HOLMES, Brunswick, Maine.

Choosing a_1a_2, a common perpendicular to the lines a_1, a_2 for the axis of x, its middle point O for the origin of orthogonal coördinates, so that $OQ=x$, $PQ=y$, and denoting $Oa_1=Oa_2$ by a, and $OA_1=OA_2$ by b, we have $PA_1=\sqrt{[y^2+(b+x)^2]}$, $PP_1=a+x$, $PA_2=\sqrt{[y^2+(b-x)^2]}$, $PP_2=a-x$.



From the condition of the problem

$$\frac{\sqrt{[y^2+(b+x)^2]}}{a+x}=\frac{\sqrt{[y^2+(b-x)^2]}}{a-x}.$$

Squaring, clearing of fractions, and simplifying, we finally and without difficulty obtain the equation

$$\frac{y^2}{b(a-b)}+\frac{x^2}{ab}=1.$$

If $a>b$, that is, for the case that A_1 and A_2 are situated within the parallels a_1 and a_2, the equation is that of an ellipse, whose foci are A_1 and A_2, semi-axes $\sqrt{(ab)}$, and $\sqrt{[b(a-b)}$.

If $a<b$, that is, for the case that A_1 and A_2 lie outside of the parallels a_1 and a_2, the curve is an hyperbola.

Also solved by G. B. M. Zerr, L. E. Newcomb, and G. W. Greenwood.

---

*$AN=\frac{3}{4}AD$, $NG=\frac{1}{4}AD$; $AM=\frac{9}{16}AD$, $MK=\frac{3}{16}AD$; hence $AN/NG=AM/MK$. Ed.